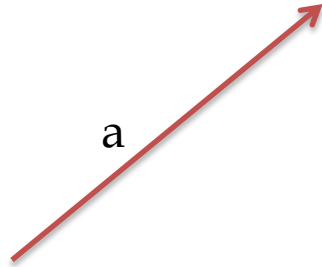# *Computer Graphics*

Lecture 20

Vector
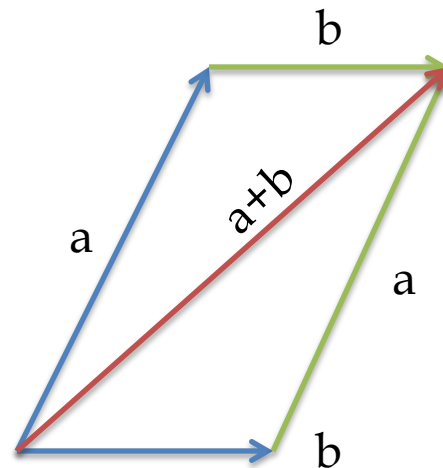
A vector is a value that describes both a magnitude and direction.

Vector Arithmetic

To find the SUM of two vectors we place the tail of one to the head of the other That complete the triangle.
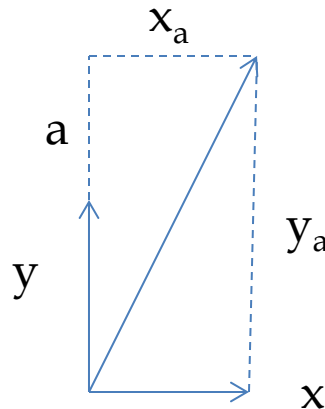
a+b = b+a

Vector Coordinate

Vectors Expressed by orthonormal coordinates

a = (Xa, Ya)

Magnitude can be calculate as

$$\|a\| = \sqrt{x_a{}^2 + ya^2}$$

Dot Product
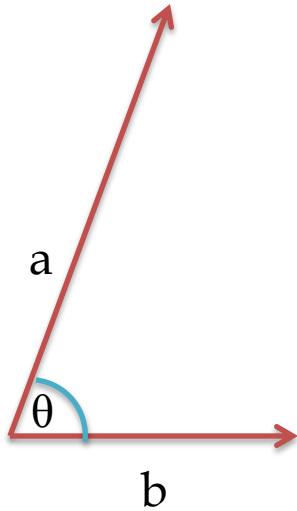
Dot Product define an

a.b = ||a|| ||b|| $\cos\theta$
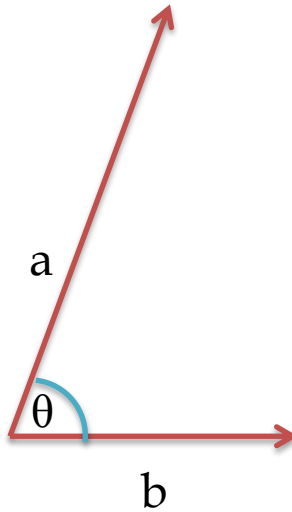
Where θ is the angle between the two vectors

a

θ

b

Cross Product

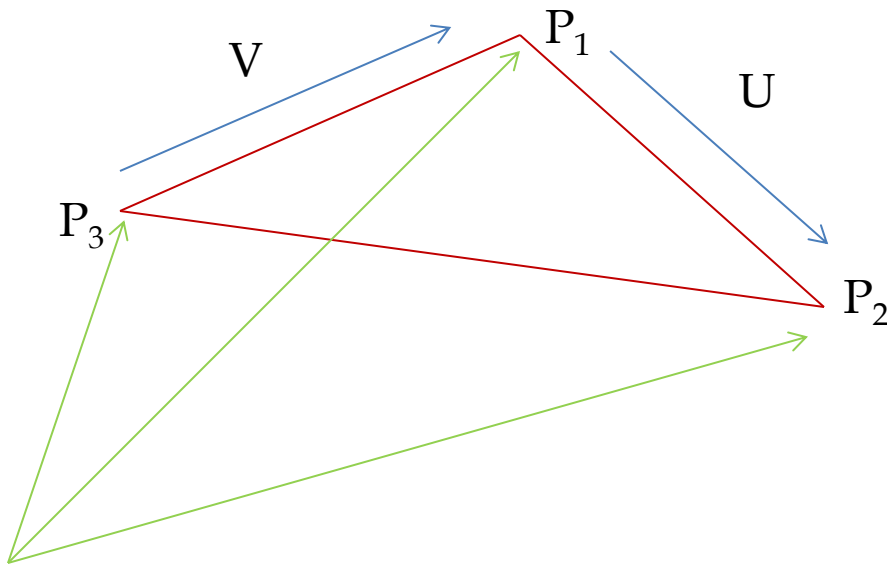Cross product is another vector multiplication operation.

Usually used only for 3D vectors.

a x b = ||a||||b||sonθ

Calculating Normal

A surface normal for a triangle can be calculated by taking the vector cross product of two edges of that triangle.

V

P_1

U

P_3

P_2

So for a triangle p1, p2, p3,

if the vector U = p2 - p1 and
the vector V = p1 – p3

then the normal $N = U \, X \, V$
(anti-clockwise order)

Nx, Ny, Nz can be calculated by

Nx = UyVz - UzVy

Ny = UzVx - UxVz

Nz = UxVy – UyVx

Length of the vector can be calculated as

$$l = \sqrt{Nx^2 + Ny^2 + Nz^2}$$

Normalization

Normalization implies first computing the length of the vector, and then dividing each component by the vectors length.
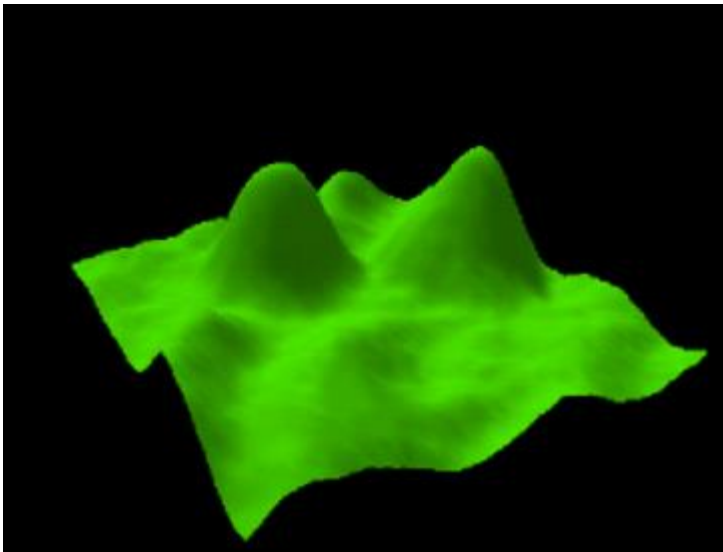
$$Nvx = Nx/ \sqrt{Nx^2 + Ny^2 + Nz^2}$$

$$Nvy = Ny/ \sqrt{Nx^2 + Ny^2 + Nz^2}$$

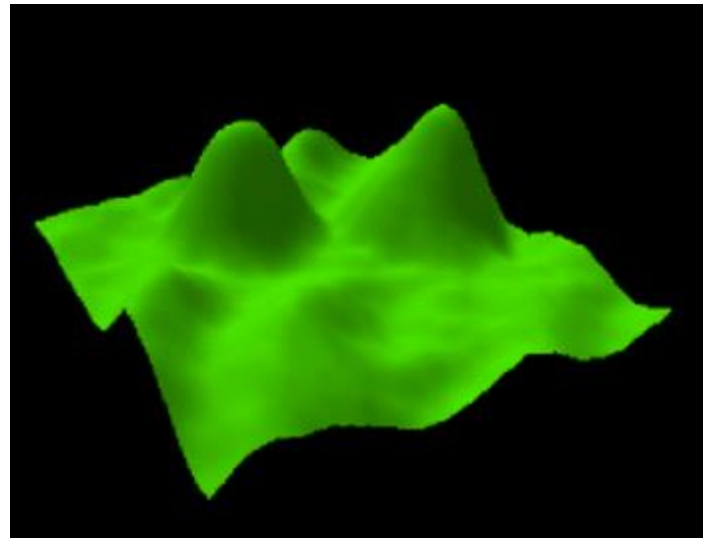$$Nvz = Nz/ \sqrt{Nx^2 + Ny^2 + Nz^2}$$

Vertex Normal

The main problem with assigning a normal per face is that the terrain may looks faced when it comes to low polygons.
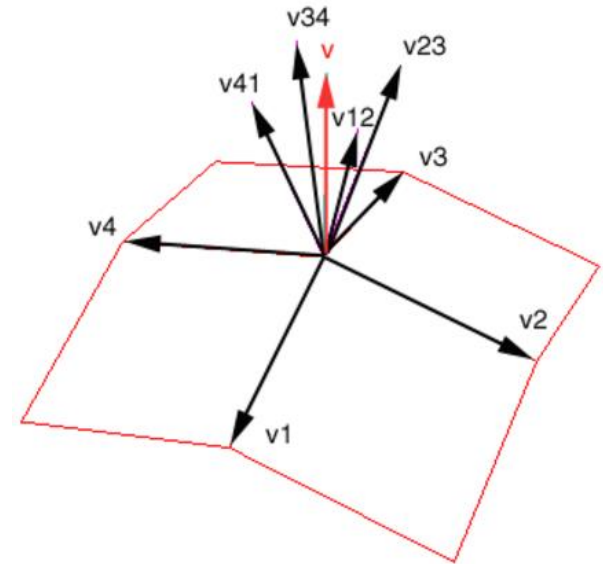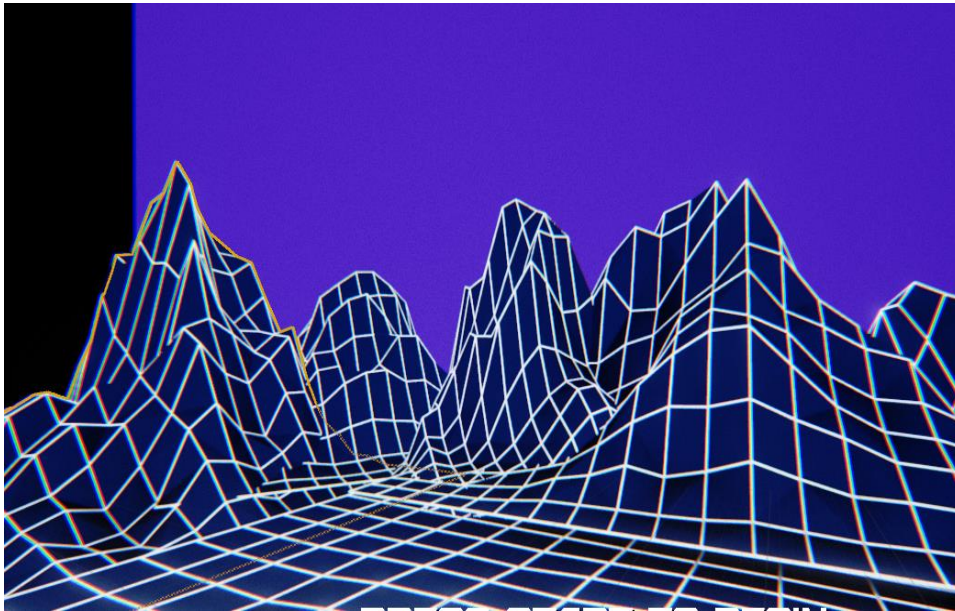

Rough

Smooth

In order to get a smoother look normal should be computed per vertex, and not per face.

When computing normal per vertex it is necessary to take into account the faces that share the vertex.

The normal at a vertex should be computed as the normalized sum of all the unit length normal for each face the vertex shares.

Vertex Normal v = ($\sum$ v12, v23, v34, v41) / $\sqrt{v_1{}^2 + v_2{}^2 + v_3{}^2 + v_4{}^2}$

Barycentric Coordinates

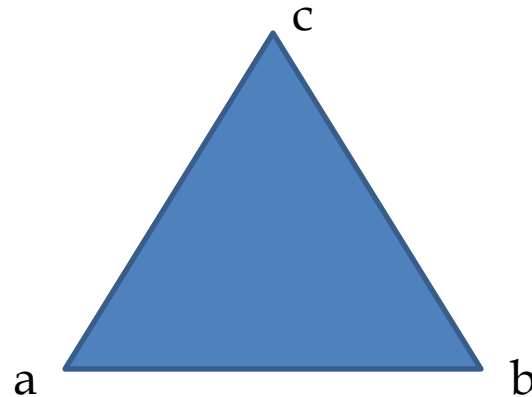Triangles are the fundamental primitive used in 3D modeling programs.

We can express any point p coplanar to the Triangle as

$P = a + \beta(b\text{-}a) + \lambda(c\text{-}a)$

We rewrite this as

$P(\alpha,\beta,\lambda) = \alpha a + \beta b + \lambda c$ *where*
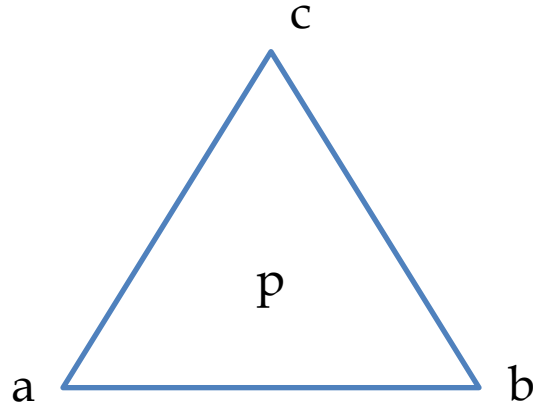
$\alpha \equiv 1 - \beta - \lambda$

Example

Let

$a = p(1,0,0)$

$b = p(0,1,0)$

$c = p(0,0,1)$

Point p is inside the triangle if
$0 < \alpha < 1$
$0 < \beta < 1$
$0 < \lambda < 1$

❖ If one component is zero
   o P is on an edge
❖ If two components are zero
   o P is on a vertex